Exam Statistical Physics II
Friday, June 17, 2011

1. PROBLEM 1: ENERGY-ENERGY CORRELATIONS IN THE ISING MODEL (10 MARKS)

Consider an Ising model with vanishing external field ($H = 0$) in d dimensions and introduce the quantity

$$E_i = -\frac{1}{2} J \sum_{\langle i,j \rangle} s_i s_j \ ,$$

where the sum is taken at fixed i.

- Show that the specific heat of the considered Ising system can be derived from the following expression

$$C_V = k_B \beta^2 \sum_{i,j} \langle (E_i - \langle E_i \rangle)(E_j - \langle E_j \rangle) \rangle \ .$$

  Discuss clearly over which variables the sum $\sum_{i,j}$ extends.

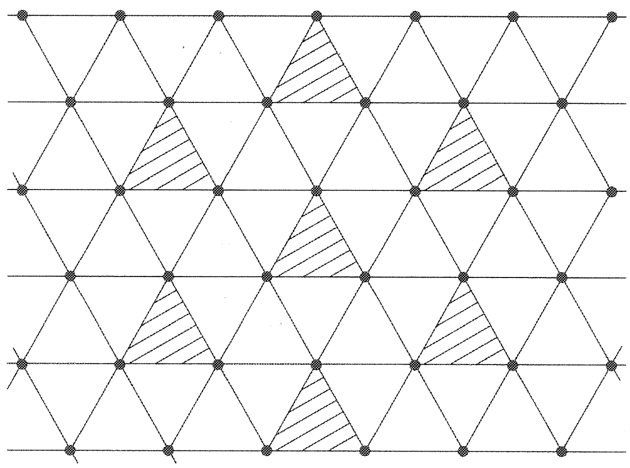
- Compute explicitly $\langle (E_i - \langle E_i \rangle)(E_j - \langle E_j \rangle) \rangle$ for the one-dimensional case. HINT: You may find it appropriate to tompute the so-called four-point correlation function of the one-dimensional Ising system: $\langle s_i s_j s_k s_l \rangle$ ($i \leq j \leq k \leq l$).

- Check your results by computing C_V directly from one of the exact solutions of the one-dimensional Ising model.

2. PROBLEM 2 (10 MARKS)

- Consider one-dimensional site percolation: use the concept of scale invariance in order to determine the p_c and the critical exponent corresponding with the "connectedness length".
- Consider two-dimensional site percolation on a triangular lattice

Tip: zie opl 9.10



Compute the p_c and the critical exponent ν.

- Consider two-dimensionsal site percolation on a square lattice. The standard renormalization method consists of reducing cells of dimension $n \times n$ to cells of dimension 1. The corresponding renormalization function is denoted by $R(p, n)$.

Prove the following expression for the critical exponent ν

opl 9.12

$$\nu = \frac{\ln \frac{n_1}{n_2}}{\ln \frac{\lambda_{n_1}}{\lambda_{n_2}}} ,$$

with

$$\lambda_n = \left. \frac{dR(p,n)}{dp} \right|_{p=p*(n)} .$$

- The critical exponents for percolation satisfy a set of relations that are identical to the scaling relations for thermal systems

$$2\beta + \gamma = d\nu ,$$

opl 9.5

where d is the spatial dimension. Make this expression plausible.