eerste zittijd 2010-2011

Exam Logic

Problem 1: Determine the cardinality of the set of continuous real functions! (Hint: Consider the set of functions $f : \mathbb{Q} \to \mathbb{R}$) (2 points)

Problem 2: Let $(a_i)_{i=1}^k$, $(b_i)_{i=1}^\ell$ be two finite sequences of integers. Put $(a_i)_{i=1}^k < (b_i)_{i=1}^\ell$, if either $k < \ell$, and $a_i = b_i$ for $1 \le i \le k$, or there is some $i_0 < \min(k, \ell)$, such that $a_{i_0} < b_{i_0}$ and $a_i = b_i$ for $1 \le i < i_0$. Show that the set of all finite sequences together with this order does not form a well order. Show that the set of all strictly decreasing finite sequences does form a well order. (4 points)

Problem 3: Let G be a finitely generated group, H a subgroup of G. Show that there exists a maximal proper subgroup of G containing H, that is, a subgroup H' such that $H \subseteq H' \subset G$. Give an example of a not finitely generated group, for which this statement is false!(3 points)

Problem 4: Let L be a countable language, T an L-theory. Suppose there exists some cardinality κ, such that T has precisely one model up to isomorphy. Show that T is complete! (Hint: use Löwenheim-Skolem) (3 points)

$|M| = n$

Problem 5: Let L be a language, φ a formula. The *spectrum* of φ is the set of all integers n, such that there exists an L-structure M with $M \models \varphi$. A set of integers A is called a spectrum, if there exists a formula φ with spectrum A.

(1) Proof that every finite set of integers is a spectrum. (2 points)
(2) Proof that the set of even integers is a spectrum. (2 points)
(3) Does there exist a set of inte>gers, which is not a spectrum? (2 points)

Problem 6: Give a finite automaton that recognizes a PIN! More precisely, the automaton reads 4 digits in $\{0, 1, \ldots, 9\}$, and ends in one of two states (accept, don't accept). Try to be economic and not to use too many states! (2 points)

Duration: 3 hours

The grade of this exam is computed from the points obtained from the problems using the function which maps 0 to 0, 2.5 to 5, 12.5 to 15, 20 to 20, and is linear inbetween. The final grade is computed from the grade of this exam and the grade of the project weighted 2:1.